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THE DAMPING OF THE HORIZONTAL BETATRON OSCILLATIONS  
OF AN OFF-MOMENTUM BUNCH

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## I. Introduction

Due to the possible mismatch during injection which will lead to betatron oscillations and eventual dilution of the beam, super-fast transverse dampers<sup>1</sup> are designed and installed in the Energy Saver. Each damper consists of a kicker which kicks the bunch according to its transverse displacement from the ideal orbit monitored by a pickup situated ideally about a quarter betatron wavelength upstream. However, an injected bunch will contain also momentum error which will place the bunch on an off-momentum orbit. As a result, even when there is no betatron oscillation, the horizontal pickup will monitor a non-zero displacement of the bunch from the ideal orbit and the horizontal kicker downstream will kick it and therefore create betatron oscillations. Although the off-momentum function is known at the horizontal pickup, the off-momentum orbit will oscillate about the designed orbit when the RF is switched on. For the Energy Saver, it takes only about 170 turns for this orbit to oscillate from zero displacement to its maximum. On the other hand, the horizontal damper is also designed to damp out betatron oscillations in about 100 turns. Thus, it is quite hard for the pickup to tell whether a beam displacement is due to betatron oscillations or off-momentum displacement. Because of this, we are faced with the problem: Will such a damper damp out the betatron oscillations of an off-momentum bunch eventually? This problem will be

examined in this note. The result shows that the damper will damp the bunch to a new perturbed off-momentum orbit which is displaced from the original off-momentum orbit by  $O(\frac{1}{2}\zeta\beta)$ , where  $\beta$  is the beta-function at the pickup and

$$\zeta = \frac{eV}{E_0} \frac{l}{d} \quad (1)$$

is a parameter of the kicker whose length is  $l$ , gap is  $d$  and gap voltage is  $V$  per unit displacement at the pickup. In the case of the designed damper in the Energy Saver,

$$\begin{aligned} \beta &\approx 10^4 \text{ cm}, \\ l &\approx 120 \text{ cm}, \\ d &\approx 5 \text{ cm}, \\ V &\approx 100 \text{ volts/cm}. \end{aligned} \quad (2)$$

Thus  $\frac{1}{2}\zeta\beta \sim 8\%$  for beam energy  $E_0 = 150 \text{ GeV}$ . When the kicker is turned off, the bunch starts betatron oscillations around the original off-momentum orbit with amplitude  $\sim O(\frac{1}{2}\zeta\beta)$  of the off-momentum displacement. However, if  $V$  is switched off adiabatically (Section IV), the bunch can be brought to its exact off-momentum orbit without any betatron oscillations.

In Section V, we are going to discuss a damper system where the signal from the pickup passes through a filter of narrow bandwidth so that it monitors only the betatron oscillation displacement regardless of whether the particle is in an off-momentum orbit.

## II. Derivation

The fact that the off-momentum orbit oscillates when the RF is switched on leads us to a study of the coupled betatron-synchrotron motion. We assume that the bunch under consideration is small enough to be represented

by a point. The coupled horizontal motion can then be described by four coordinates  $x$ ,  $x'$ ,  $z$  and  $\delta$ , where  $x$  and  $z$  are the horizontal and longitudinal displacements of the bunch center relative to the ideal designed trajectory,  $x'$  is the angle between bunch's direction of motion and the ideal trajectory and  $\delta = \Delta E/E$  is the relative energy error of the bunch.

The equations of motion governing these coordinates are

$$\begin{aligned} x'' + Gx - K\delta &= 0, \\ z' + Kx &= 0, \\ \delta' - \frac{1 - \cos 2\pi\nu_s}{\pi\alpha R} \cdot z \delta(s-s_c) &= 0, \end{aligned} \quad (3)$$

where  $K(s)$  is the curvature of the trajectory,  $G(s)$  the guide field function provided by the quadrupoles,  $\nu_s$  the synchrotron tune, (which is the number of synchrotron oscillations per turn),  $R$  the radius of the ring,  $\alpha = -(\Delta\omega/\omega)/(\Delta p/p)$  the ratio of the fractional change in revolution frequency to the fractional change in momentum of a point particle. The differentiations are with respect to the longitudinal position  $s$ . In above, we have assumed that there is only one RF cavity located at  $s_c$ . For the case of many cavities at  $s_i$ ,  $\delta(s-s_c)$  should be replaced by  $\sum_i \delta(s-s_i)$ .

When the bunch deviates a horizontal distance  $x_m$  from the ideal trajectory, the kicker will change its angle by an amount

$$\Delta x' = \zeta x_m. \quad (4)$$

In the four coordinates  $x$ ,  $x'$ ,  $z$  and  $\delta$  are written as a column matrix<sup>2</sup>, this change in  $x'$  can be represented by a transfer matrix from the monitor to the kicker:  $\zeta J$ , with

$$J = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (5)$$

The transfer matrix for one complete revolution of the ring starting from the exit of the RF cavity is given by

$$T_{tot} = T_{cav} T_0 + S T_{cav} T_{ck} J T_{mc}, \quad (6)$$

where

$$T_{cav} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1 - \cos 2\pi \nu_s}{\pi \alpha R} & 1 \end{pmatrix} \quad (7)$$

is the transfer matrix across the RF cavity,  $T_0$  is the transfer matrix for the rest of the ring (with the damper omitted),  $T_{mc}$  and  $T_{ck}$  are the transfer matrices from cavity to monitor and kicker to cavity respectively.

Explicitly, these matrices are given by

$$T_0 = \begin{pmatrix} r_{11} & r_{12} & 0 & \eta_c - r_{11} \eta_c' - r_{12} \eta_c' \\ r_{21} & r_{22} & 0 & \eta_c' - r_{21} \eta_c - r_{22} \eta_c' \\ a & b & 1 & -2\pi \alpha R - a \eta_c - b \eta_c' \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (8)$$

$$T_{mc} = \begin{pmatrix} R_{11} & R_{12} & 0 & \eta_m - R_{11} \eta_c - R_{12} \eta_c' \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{pmatrix}, \quad (9)$$

and

$$T_{ck} = \begin{pmatrix} - & \bar{R}_{12} & - & - \\ - & \bar{R}_{22} & - & - \\ - & \bar{B} & - & - \\ - & 0 & - & - \end{pmatrix}. \quad (10)$$

In  $T_{mc}$  and  $T_{ck}$ , only the elements relevant to our deviation are displayed.

In above, we have defined

$$r_{11} = \cos 2\pi\nu_k + \alpha_c \sin 2\pi\nu_k,$$

$$r_{12} = \beta_c \sin 2\pi\nu_k,$$

$$r_{21} = -\frac{1+\alpha_c^2}{\beta_c} \sin 2\pi\nu_k,$$

$$r_{22} = \cos 2\pi\nu_k - \alpha_c \sin 2\pi\nu_k,$$

$$a = r_{21}\gamma_c - r_{11}\gamma_c' + \gamma_c',$$

$$b = -r_{12}\gamma_c' + r_{22}\gamma_c - \gamma_c,$$

$$R_{11} = (\beta_m/\beta_c)^{\frac{1}{2}} [\cos(\phi_m - \phi_c) + \alpha_c \sin(\phi_m - \phi_c)],$$

$$R_{12} = (\beta_m\beta_c)^{\frac{1}{2}} \sin(\phi_m - \phi_c),$$

$$\bar{R}_{12} = (\beta_c\beta_k)^{\frac{1}{2}} \sin(2\pi\nu_k + \phi_c - \phi_k),$$

$$\bar{R}_{22} = (\beta_k/\beta_c)^{\frac{1}{2}} [\cos(2\pi\nu_k + \phi_c - \phi_k) - \alpha_c \sin(2\pi\nu_k + \phi_c - \phi_k)],$$

$$\bar{B} = -\bar{R}_{12}\gamma_c' + \bar{R}_{22}\gamma_c - \gamma_k,$$

$\eta_{c,m,k}$  = off-momentum functions at cavity, monitor and kicker,

$\beta_{c,m,k}$  = horizontal beta-functions at cavity, monitor and kicker,

$\alpha_c$  = horizontal alpha-function at cavity,

$\phi_{c,m,k}$  = horizontal betatron phases with  $2\pi\nu_x > \phi_k > \phi_m > \phi_c > 0$ ,

$\nu_x$  = horizontal betatron tune.

The horizontal motion of the bunch is governed by the eigenvalues  $\lambda$  of the transfer matrix  $T_{tot}$ , which are solutions of the secular equation

$$\det(T_{tot} - \lambda) = 0. \quad (11)$$

In order to simplify the derivation, the pickup is placed at the exit of the RF cavity. Equation (11) then turns out to be

$$\begin{aligned} & (1 - 2\lambda \cos 2\pi\nu_x + \lambda^2)(1 - 2\lambda \cos 2\pi\nu_s + \lambda^2) \\ & - \frac{(1 - \cos 2\pi\nu_s) \sin 2\pi\nu_x}{\pi \alpha_R \beta_c} [\eta_c^2 + (\alpha_c \eta_c + \beta_c \eta'_c)^2] \lambda (1 - \lambda)^2 \\ & - 5(\beta_k \beta_c)^{\frac{1}{2}} (1 - \lambda)^2 [\sin(\phi_k - \phi_c) + \lambda \sin(2\pi\nu_x + \phi_c - \phi_k)] \\ & + 5 \frac{2\pi\nu_s^2}{\alpha_R} \lambda (1 - \lambda) [\bar{R}_{12} \eta_c (r_{22} \eta'_c + r_{21} \eta_c - \eta'_c) + \bar{R}_{22} \eta_c (\eta_c - r_{11} \eta_c - r_{12} \eta'_c) \\ & - \eta_k (\eta_c - r_{11} \eta_c - r_{12} \eta'_c)] + 5 \frac{4\pi\nu_s^2}{\alpha_R} \eta_c \eta_k (1 - \cos 2\pi\nu_x) = 0, \end{aligned} \quad (12)$$

where we have left out terms of order higher than  $O(\zeta v_s^2)$ . The first term on the left side of Eq. (12) is the unperturbed and therefore uncoupled contribution. The second term is the synchro-betatron coupling contribution. The rest, being proportional to  $\zeta$ , gives the linear contribution of the damper. The eigenvalues can be expressed formally as

$$\lambda_k = \exp \left[ -\alpha_k \pm i 2\pi (\nu_k + \Delta \nu_k) \right], \quad (13)$$

where  $k = x, s$  stands for betatron or synchrotron respectively. Thus,  $\alpha_k$  represents the damping per turn and  $\nu_k$  the tunes of the corresponding oscillation. After obtaining  $\lambda$ , the horizontal transverse position of the bunch after  $n$  revolutions can be written exactly as

$$\begin{aligned} x_n &= e^{-n\alpha_x} \left[ A \cos 2\pi n \nu'_x + B \sin 2\pi n \nu'_x \right] \\ &\quad + e^{-n\alpha_s} \left[ (x_0 - A) \cos 2\pi n \nu'_s + C \sin 2\pi n \nu'_s \right], \\ x'_n &= e^{-n\alpha_x} \left[ A' \cos 2\pi n \nu'_x + B' \sin 2\pi n \nu'_x \right] \\ &\quad + e^{-n\alpha_s} \left[ (x'_0 - A') \cos 2\pi n \nu'_s + C' \sin 2\pi n \nu'_s \right], \end{aligned} \quad (14)$$

where the perturbed betatron tune and synchrotron tune are

$$\nu'_x = \nu_x + \Delta \nu_x \quad \text{and} \quad \nu'_s = \nu_s + \Delta \nu_s. \quad (15)$$

The parameters  $A, B, C$  and  $A', B', C'$  are functions of the initial coordinates  $x_0, x'_0, z_0$  and  $\delta_0$ . They can be obtained by diagonalizing the transfer matrix  $T_{\text{tot}}$ . Under the assumption that  $\nu_s \ll 1$ ,  $\Delta \nu_k \ll 1$  and  $\alpha_k \ll 1$ , we obtain explicitly



$$\begin{aligned}
 \alpha_x &= \frac{1}{2} \zeta (\beta_c \beta_k)^{\frac{1}{2}} \sin \phi, \\
 2\pi \Delta \nu_k &= -\frac{1}{2} \zeta (\beta_c \beta_k)^{\frac{1}{2}} \cos \phi, \\
 \alpha_s &= O(\zeta \beta \nu_s^2), \\
 2\pi \Delta \nu_s &= \zeta \nu_s \gamma_k \gamma_c / 2\alpha R,
 \end{aligned} \tag{16}$$

and

$$\begin{aligned}
 A &= x_0 - \bar{\gamma}_c \delta_0, \\
 B &= [\alpha_c(x_0 - \bar{\gamma}_c \delta_0) + \beta_c(x'_0 - \bar{\gamma}'_c \delta_0)] \left[ 1 + \frac{1}{2} \zeta (\beta_c \beta_k)^{\frac{1}{2}} \cos \bar{\phi} / \sin 2\pi \nu_k \right] \\
 &\quad + \frac{1}{2} \zeta (\beta_c \beta_k)^{\frac{1}{2}} (x_0 - \bar{\gamma}_c \delta_0) \sin \bar{\phi} / \sin 2\pi \nu_k, \\
 C &= \nu_s \gamma_c z_0 / \alpha R, \\
 A' &= x'_0 - \bar{\gamma}'_c \delta_0, \\
 B' &= -\left[ (1 + \alpha_c^2)(x_0 - \bar{\gamma}_c \delta_0) / \beta_c + \alpha_c(x'_0 - \bar{\gamma}'_c \delta_0) \right] \left[ 1 + \frac{1}{2} \zeta (\beta_c \beta_k)^{\frac{1}{2}} \cos \bar{\phi} / \sin 2\pi \nu_k \right] \\
 &\quad + \frac{1}{2} \zeta (\beta_c \beta_k)^{\frac{1}{2}} \left[ 2(x_0 - \bar{\gamma}_c \delta_0)(\cos \bar{\phi} - \alpha_c \sin \bar{\phi}) / \beta_c \sin 2\pi \nu_k \right. \\
 &\quad \left. - (x'_0 - \bar{\gamma}'_c \delta_0) \sin \bar{\phi} / \sin 2\pi \nu_k \right], \\
 C' &= \nu_s \gamma'_c z_0 / \alpha R,
 \end{aligned} \tag{17}$$

where we have used

$$\phi = \phi_k - \phi_c, \quad (18)$$

$$\phi = 2\pi\nu_k - \phi, \quad (19)$$

$$\bar{\eta}_c = \eta_c + \frac{1}{2} \zeta (\beta_c \beta_k)^{\frac{1}{2}} \eta_c (\sin \phi + \cos \phi \cot \pi \nu_k), \quad (20)$$

$$\bar{\eta}_c' = \eta_c' + \frac{1}{2} \zeta (\beta_k / \beta_c)^{\frac{1}{2}} \eta_c' [(\sin \phi - \alpha_c \cos \phi) \cot \pi \nu_k - \cos \phi - \alpha_c \sin \phi]. \quad (21)$$

We note that the magnitude of  $C$  is  $O(\eta \delta_0)$  and that of  $C'$  is  $O(\eta' \delta_0)$ ; the next higher order is rather hard to compute.

We see that with a proper choice of the phase advance  $\phi$  from the pickup to the kicker and the sign of the kicker voltage, all betatron oscillations will be damped when  $n$  is big enough leaving behind only synchrotron oscillation. At the pickup, if the bunch is placed initially at

$$x_0 = \bar{\eta}_c \delta_0,$$

$$x_0' = \bar{\eta}_c' \delta_0,$$

it will come back to the same position turns after turns when the RF is switched off. Thus,  $\bar{\eta}_c$  and  $\bar{\eta}_c'$  become the new off-momentum functions which differ from the original ones by  $O(\frac{1}{2} \zeta \sqrt{\beta_c \beta_k})$ . The new off-momentum functions elsewhere can be obtained by similarity transformations. The new off-momentum orbit (after all betatron oscillations are damped) actually oscillates about the original orbit (with the kicker turned off) and deviates from it by  $O(\frac{1}{2} \zeta \sqrt{\beta \beta_k} \eta \delta_0)$  where  $\beta$  is the beta-function and  $\eta$  the off-momentum function at the point concerned. This new orbit is a closed one (when the RF is switched off) with a cusp at the kicker. This cusp is provided by the kicker which contributes to the bunch a sudden change in  $x'$ . In other words, we may say that the bunch actually executes betatron oscillations about the original

orbit and this betatron-oscillating trajectory happens to be a closed one due to the discontinuity imparted by the kicker, (Figure 1). As a result, when the kicker is switched off now, the bunch will pass the kicker without the cusp but continue onto a smooth trajectory which is in fact betatron oscillation about the original off-momentum orbit with amplitude of  $O(\frac{1}{2}\sqrt{\beta\beta_k}\eta\delta_0)$ .

### III. Computer Simulations

Computer simulations are made for a 150 GeV-bunch of width 0.5 cm, consisting of 300 particles and carrying an off-momentum factor of  $\delta = 0.001$ . The off-momentum functions at the pickup are  $\eta_k = 250.00$  cm and  $\eta'_k = 0.0$ . We first place the center of the bunch at  $x_0 = 0.2$  cm,  $x'_0 = 0.0$  and  $z_0 = 0.0$  cm at the pickup and observe its new position turns after turns when the damper is off. The betatron tune and synchrotron tune are taken to be  $\nu_x = 19.381$  and  $\nu_s = 0.0025$ . The results are plotted in Figure 2 as solid dots for every 5 turns. We see that the bunch executes betatron oscillations as well as synchrotron oscillations without damping. We next place the bunch at  $x_0 = 0.6$  cm,  $x'_0 = 0.0$  and  $z_0 = 0.0$  and switch on the damper. The kicker has a gain of 100 kV per cm displacement at the pickup and is placed at a phase advance of  $9.25 \times 2\pi$  from the pickup. Thus we expect to see a damping constant of  $\alpha_x = 0.08$  from Eqs. (1) and (16), where both  $\beta_c$  and  $\beta_k$  are taken as  $10^4$  cm. However, as in the actual performance of the damper, we put in a maximum gap voltage of only 8 kV across the kicker plates. As a result, the effective damping constant  $\alpha_x$  is much less. The simulations are plotted as open circles in Figure 2. We see that the betatron oscillations are damped very rapidly leaving behind the RF synchrotron oscillations. The curve joining the open circles serves the purpose of following the points only; it has no physical meaning at all.

#### IV. Adiabatic Switching-off

We note that after the damper is switched off, the bunch will execute betatron oscillations with an amplitude of  $O(\frac{1}{2}\xi\beta\eta\delta_0)$  which is directly proportional to the kicker voltage. The smaller the voltage, the smaller will be the betatron amplitude and the bunch will be left closer to the off-momentum trajectory. However, the time taken to damp out the original betatron oscillations will be considerably larger. Theoretically, if the kicker operates at a high voltage for a while and is then turned off adiabatically, the bunch will be left in the off-momentum trajectory exactly without any betatron oscillations at all. To demonstrate such a result, it is best to have the RF turned off. The kicker is allowed to operate at a gain of 100 kV per cm displacement at the pickup (maximum gap voltage being 8 kV) for 80 turns and then switched off linearly to zero in 500 turns. The bunch is centered initially at  $x_0 = 0.6$  cm,  $x'_0 = 0$  and  $\delta_0 = 0.001$  at the pickup where the off-momentum functions are  $\eta = 231.51$  cm and  $\eta' = 0.026$ . The  $\alpha$ - and  $\beta$ -functions there are taken as  $\alpha = 0.0$  and  $\beta = 10^4$  cm. The kicker is situated at a phase angle of  $0.251 \times 2\pi$  downstream and  $\beta_k$  is taken as  $10^4$  cm. The simulation results are plotted in Figure 3 for every 5 turns. The ordinate is

$$r = \left\{ \frac{(x - \eta\delta_0)^2 + [\alpha(x - \eta\delta_0) + \beta(x' - \eta'\delta_0)]^2}{[\eta^2 + (\alpha\eta + \beta\eta')^2]\delta_0^2} \right\}^{\frac{1}{2}}, \quad (22)$$

which is the fractional deviation of the bunch center from the off-momentum trajectory in the betatron-oscillation phase space. The open circles are the situation of operating the damper at the full gain of 100 kV per cm for 580 turns and then turning it off suddenly. The residual deviation is  $r = 0.47$  showing a rather big betatron amplitude left behind. The solid dots are the situation of switching off the kicker gain linearly at the 80th turn and

reaching zero at the 580th turn. The residual  $r$  is only 0.02 which is very much smaller than that by operating the damper at a full gain.

If the gap voltage of the kicker is unlimited (i.e., it can go higher than the maximum of 8 kV), we get at the end of 580 turns instead,  $r = 0.062$  for the case of operating at full gain for 580 turns and  $r = 0.000061$  for the case of switching off adiabatically. Again the latter is more efficient than the former. The former residual  $r = 0.062$  agrees with the theoretical value obtained from Eq. (22) by letting  $x = \bar{\eta}\delta_0$ ,  $x' = \bar{\eta}'\delta_0$ . The expressions for the perturbed off-momentum functions  $\bar{\eta}$  and  $\bar{\eta}'$  are given by Eqs. (20) and (21).

#### V. Pickup of Narrow Bandwidth

Another way to damp out betatron oscillations when the off-momentum displacement is not zero is to filter away all the high-frequency components and d.c. component of the pickup output.

At the azimuth  $\theta$  of the pickup, the line charge density of a charged particle is a sequence of delta-functions:

$$\lambda(t) = \frac{e}{R} \sum_{l=-\infty}^{\infty} \delta(\omega_0 t - \theta - 2\pi l), \quad (23)$$

where  $\omega_0/2\pi$  is the revolution frequency. In the positive frequency domain, this is

$$\lambda(t) = \frac{e}{2\pi R} \left[ 1 + 2 \sum_{n=1}^{\infty} \cos n(\omega_0 t - \theta) \right]. \quad (24)$$

Now consider the particle making transverse betatron oscillations with a displacement

$$x(t) = a + b \cos [\nu_x(\omega_0 t - \theta) + \phi].$$

Comparing with Eq. (14), the initial off-momentum displacement is  $a = \eta \delta_0$  (when  $\ell = 0$  in Eq. (23)) and the total initial horizontal displacement is  $x_0 = a + b \cos \phi$  where  $\phi$  is the initial phase of the betatron oscillation when the particle passes through the pickup. We note that the effects of synchrotron oscillation have been neglected here. This is because, as we shall see below, the kicker will be able to separate the betatron displacement  $b \cos[\nu_x(\omega_0 t - \theta) + \phi]$  in just a few turns which is much shorter than a synchrotron period.

The pickup responds to the line dipole density  $d$  which is given by multiplying Eqs. (24) and (25) and its expansion in frequencies is

$$\begin{aligned} d(t) = \frac{e}{2\pi R} \left\{ a + b \cos [\nu_x(\omega_0 t - \theta) + \phi] \right. \\ + 2a \sum_{n=1}^{\infty} \cos n(\omega_0 t - \theta) \\ + b \sum_{n=1}^{\infty} \cos [(n + \nu_x)(\omega_0 t - \theta) + \phi] \\ + b \sum_{n=1}^{\infty} \cos [(n - \nu_x)(\omega_0 t - \theta) + \phi] \left. \right\}. \end{aligned} \quad (26)$$

The Energy Saver has a revolution frequency of ~50 MHz and a horizontal tune of ~19.4. Thus, besides the d.c. term, the lowest frequency component is ~20 MHz coming from the last term of Eq. (26) with  $n = 20$ . The next higher one is the fundamental revolution frequency of ~50 MHz from the third term. The spectrum for low frequencies is plotted in Figure 4. We see that, if the output of the pickup is to pass through a filter of bandwidth ~15 MHz to 25 MHz, only one term of Eq. (26), i.e.

$$d(t) \Big|_{\text{filter}} = \frac{e}{2\pi R} \cdot b \cos [\tilde{\nu}_x(\omega_0 t - \theta) + \phi]$$

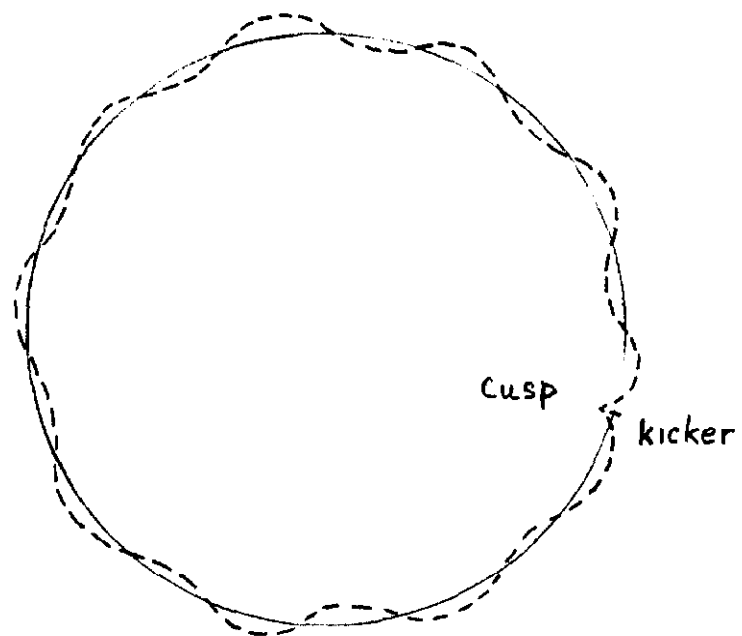
is left where  $\tilde{\nu}_x = 0.4$  which is the noninteger part of  $\nu_x$ . In fact this is

just the envelope of the response signal of the pickup with the off-momentum displacement subtracted. The number of turns which the pickup requires to sort out this bandwidth is  $\sim 50/(25-15) = 5$ . By responding to this filtered signal, the kicker can therefore damp out any betatron oscillation regardless of whether the particle is in an off-momentum orbit.

#### References

<sup>1</sup>C. Moore and R. Rice, Fermilab UPC

<sup>2</sup>A.W. Chao, P.L. Morton and J.R. Rees, PEP-Note 281, 1979



—— Original off-momentum function

----- New off-momentum function

Figure 1



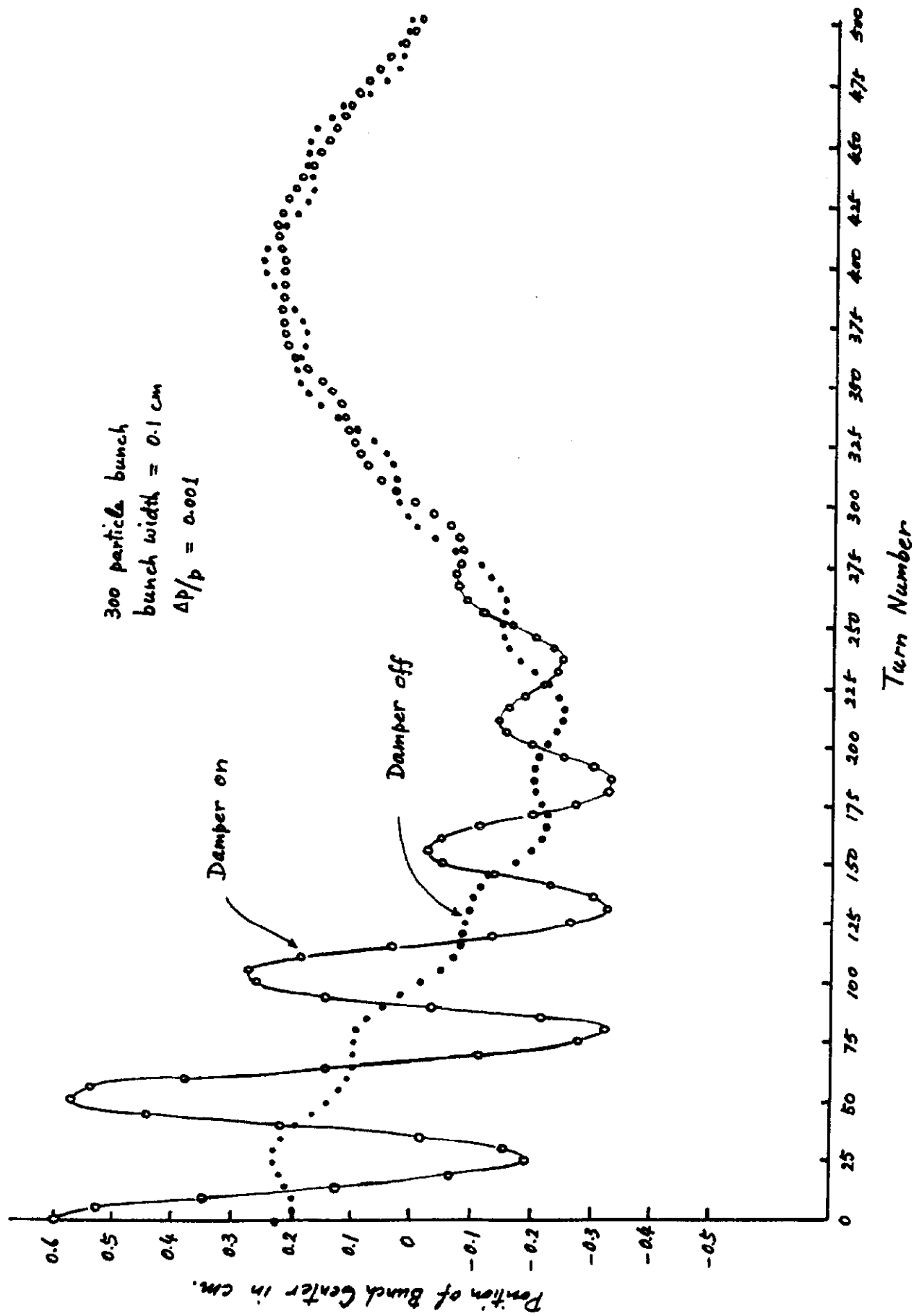


Figure 2

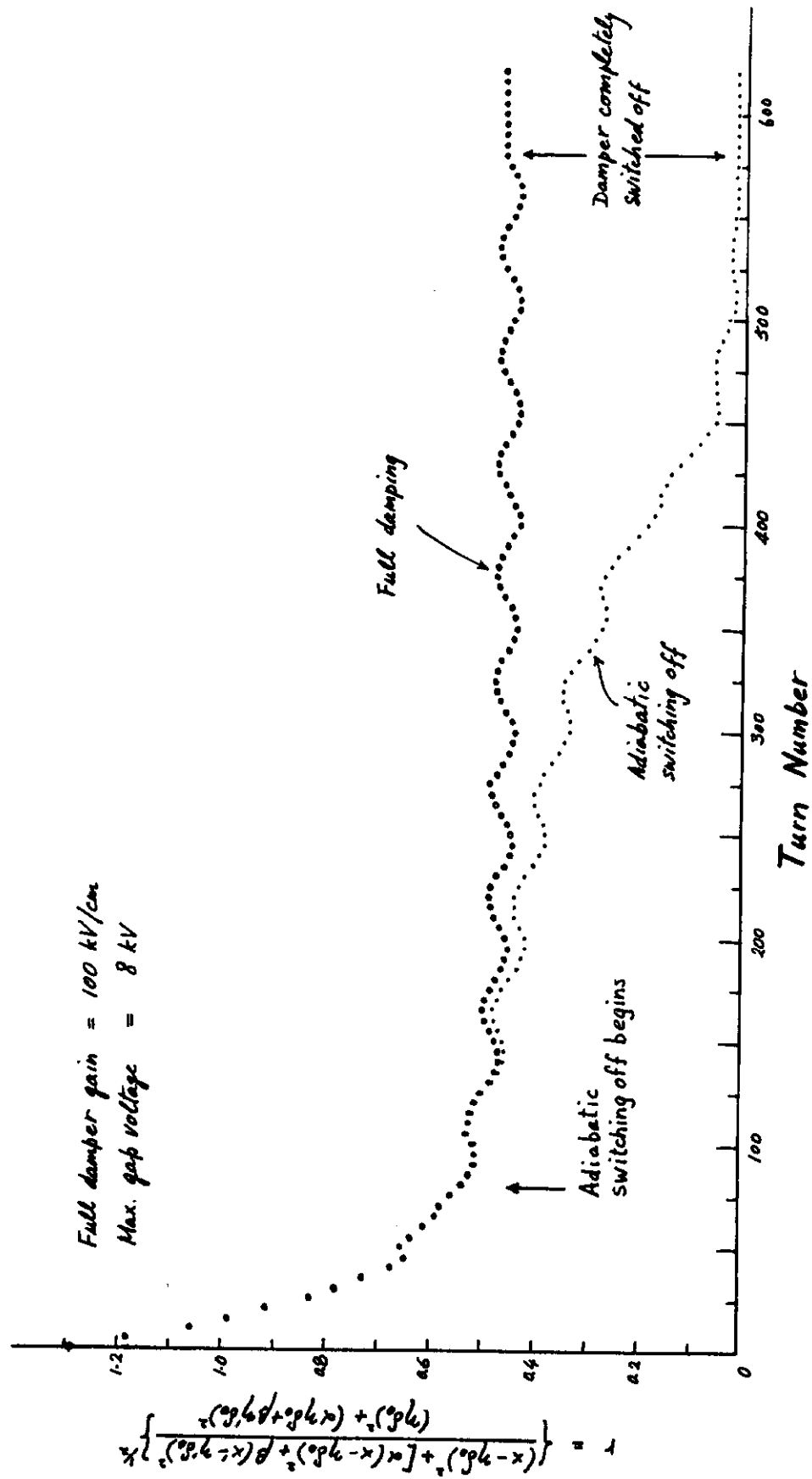


Figure 3

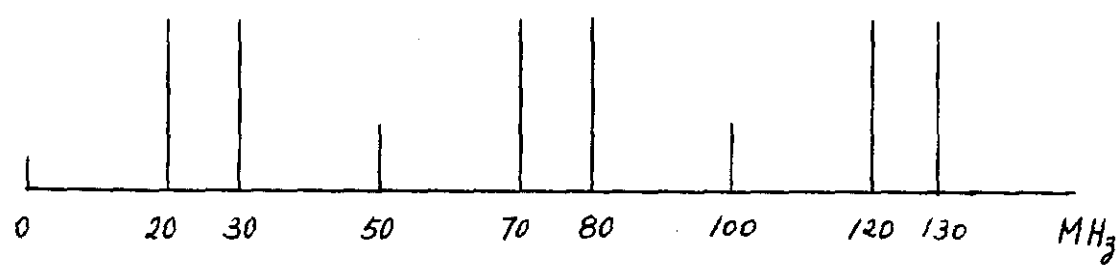


Figure 4